

## Application Note: Driving a Spring Load with a Piezoelectric Actuator

When a piezo-actuator (whether amplified or non-amplified) is actuated against a spring load, it converts electrical energy into both motion and force. This force will vary according to the amount of expansion/contraction achieved by the piezo-actuator under the applied electrical field. When activated with an applied electric field, the piezo-actuator moves against the spring load until it reaches a force balance condition. If at this point the spring load were to be removed, the stored potential energy within the piezo-actuator would be converted completely into additional motion. Therefore, when working against a spring load, the amount of displacement that the piezo-actuator can produce in the spring load is less than the piezo-actuator's free zero-load displacement. The amount of displacement that the spring can be compressed or stretched is a function of the spring stiffness and the piezo-actuator stiffness.

The following nomenclature will be used in this discussion:

### *Constants*

$k_{spr}$  : stiffness or spring constant of the applied spring load

$k_{pzt}$  : stiffness or spring constant of the piezo-actuator

$\delta_{pzt, max}$  : maximum displacement produced by the piezo-actuator with no load

$F_{max}$  : maximum force capacity produced by the piezo-actuator when  $k_{spr}$  is infinite

### *Variables*

$\delta_{pzt}$  : displacement produced by the piezo-actuator under the applied electric field

$F_{pzt}$  : force capacity produced by the piezo-actuator as it moves against a spring load

Piezo-actuators have an inverse, linear relationship between their force capacity (for pushing against a spring-type load) and their displacement. At zero displacement, an actuator has maximum force capacity ( $F_{max}$ ). At maximum displacement ( $\delta_{pzt, max}$ ) under the maximum applied rated voltage, the force capacity is zero. Similarly, when the stiffness of the spring load ( $k_{spr}$ ) is zero, the piezo-actuator is able to achieve  $\delta_{pzt, max}$ . When  $k_{spr}$  is infinite, the piezo-actuator is not able to move against the applied spring load and instead produces its maximum force capacity  $F_{max}$  against that load under the maximum rated voltage condition. The maximum force capacity  $F_{max}$  can be approximated by the product of  $\delta_{pzt, max}$  and the piezo-actuator's stiffness  $k_{pzt}$ .

One calculates the amount of stroke that the piezo-actuator can apply to the spring load using the following equations:

### Equations

The linear force-displacement relationship for a piezo-actuator pushing against a spring load is defined as

$$(1) \delta_{pzt} = \delta_{pzt,max} - \frac{\delta_{pzt,max}}{F_{max}} F_{pzt} \quad \text{or} \quad F_{pzt} = F_{max} \left( 1 - \frac{\delta_{pzt}}{\delta_{pzt,max}} \right)$$

where  $\delta_{pzt}$  is the deflection of the piezo-actuator and  $F_{pzt}$  is the corresponding force capacity that can be generated by the piezo-actuator at that deflection

The force-displacement relationship of the spring load is defined as:

$$(2) F_{spr} = \delta_{spr} k_{spr}$$

Recognizing that at equilibrium the force provided by the piezo-actuator is equal in magnitude to the opposing force from the spring load, we can equate the expressions from (1) and (2):

$$(3) F_{spr} = F_{pzt} = \delta_{spr} k_{spr} = F_{max} \left( 1 - \frac{\delta_{pzt}}{\delta_{pzt,max}} \right)$$

At force equilibrium, the actuator has compressed the spring by a distance

$$(4) \delta_{pzt} = \delta_{spr}$$

Substituting (4) into (3) yields

$$(5) \delta_{spr} = F_{max} \left( k_{spr} + \frac{F_{max}}{\delta_{pzt,max}} \right)^{-1} = \frac{F_{max}}{k_{spr} + k_{pzt}}$$

Generally, if the spring stiffness exceeds the piezo-actuator stiffness, less than one-half of the piezo-actuator's free displacement can be applied to the external spring. If the spring stiffness equals the piezo-actuator stiffness, then exactly one-half of the piezo-actuator's free displacement can be applied to the spring. If the spring stiffness is less than the piezo-actuator stiffness, more than one-half of the piezo-actuator's free displacement can be applied to the external spring. The

portion of the piezo-actuator's free displacement not applied to the external spring is stored in the piezo-actuator as potential energy.

*Examples of a piezo-actuator driving against a spring load*

FPA-100 piezo-actuator: nominal spring stiffness  $k_{pzt} = 1.0$  N/micron; nominal blocked force  $F_{max} = 145$  N. For reference, nominal displacement  $\delta_{pzt, max} = 145$  microns.

Case A: spring load having a stiffness of 0.25 N/micron (25% of the FPA-100 stiffness)

Using Equation (5), we determine  $\delta_{spr}$  :

$$\delta_{spr} = 145 \text{ N} / [(0.25 \text{ N}/\mu\text{m}) + 1 \text{ N}/\mu\text{m}] = 116 \mu\text{m}$$

Case B: spring load having a stiffness of 1.5 N/micron (150% of the FPA-100 stiffness)

Using Equation (5), we determine  $\delta_{spr}$  :

$$\delta_{spr} = 145 \text{ N} / [(1.5 \text{ N}/\mu\text{m}) + 1 \text{ N}/\mu\text{m}] = 58 \mu\text{m}$$

Case C: spring load having a stiffness of 1.0 N/micron (equivalent to the FPA-100 stiffness)

Using Equation (5), we determine  $\delta_{spr}$  :

$$\delta_{spr} = 145 \text{ N} / [(1 \text{ N}/\mu\text{m}) + 1 \text{ N}/\mu\text{m}] = 72.5 \mu\text{m}$$